PROBABILISTIC ANALYSIS OF CONCRETE BEAMS DURING FIRE

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INTRODUCTION

In this paper a simple computational tool is presented, which provides insight in the time and temperature dependent reliability of concrete beams during fire. The uncertainty of basic variables is taken into account through Monte Carlo simulations, resulting in a quantification of the uncertainty regarding the bending moment capacity during fire and the corresponding evolution of the safety level. The results of these full-probabilistic simulations are compared with the semi-probabilistic calculation methods as specified in EN 1992-1-2 (CEN, 2004a).

1 MODEL CONCEPTS

The goal of this paper is to quantify the evolution of the structural safety of concrete beams subjected to bending during fire. Due to the changing temperature distribution over the concrete cross-section, the limit state function for bending during fire cannot be formulated analytically. Hence, a computational tool was developed in order to calculate the structural response of concrete members during fire iteratively and to enable a full-probabilistic analysis of this structural response.

1.1 Deterministic analysis

A basic deterministic model is developed in EXCEL, calculating the bending moment capacity for a concrete beam at t minutes of exposure to the ISO 834 fire curve. The beam is assumed to be exposed to fire from three sides (bottom and side faces). The temperature distribution in a cross-section is calculated by the finite element program DIANA and used as input for the EXCEL model.

The bending moment capacity ($M_{R,fi,t}$) is calculated in the ultimate limit state and by the assumptions of the classical linear-elastic structural analysis according to EN 1992-1-1 (CEN, 2004b). Stresses introduced by internal thermal restraint are not considered in this model.

The effects of fire on the material properties of both concrete and reinforcing steel are considered through a temperature dependent function. This kind of simplification corresponds to the methodology followed in EN 1992-1-2. The actual evaluation of the local temperatures θ_i in the concrete cross-section and the corresponding local material properties is performed in discrete square elements measuring 5 mm x 5mm. This type of discretization is visualized in Fig. 1, with ε_i the local strain in the ultimate bending limit state, equal for all i at the same vertical distance from the beam bottom. For each discrete concrete segment, the reduction factors of EN 1992-1-2 on the material properties are applied. The model allows to implement alternative definitions of the reduction factors.

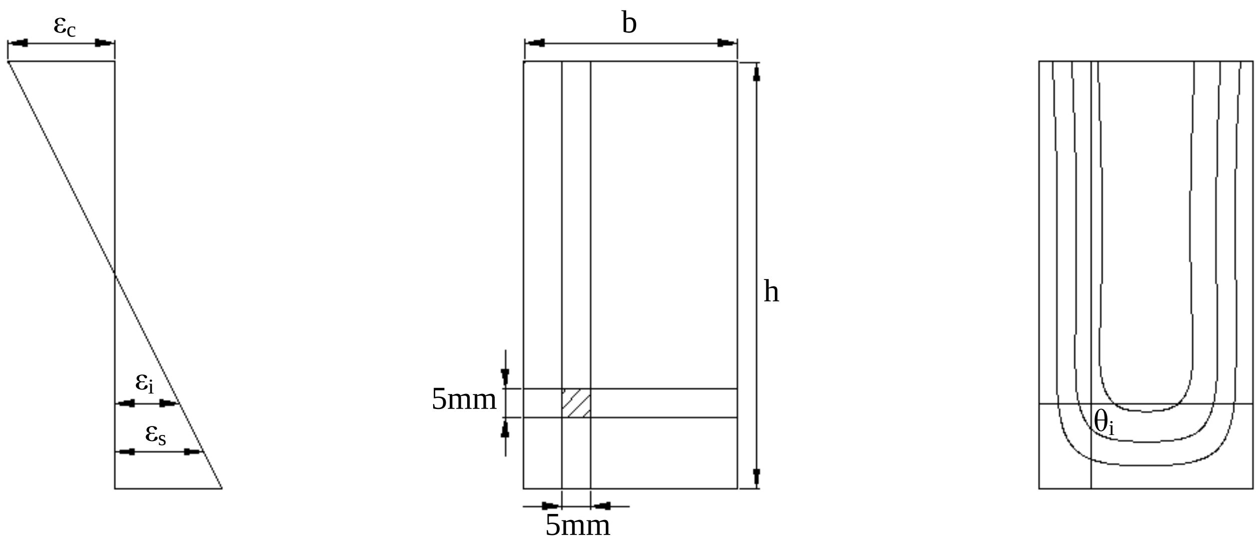

Fig. 1: Discretization of beam cross
(concrete beam (center), temperature distribution (right), distribution of strains (left))

1.2 Modelling of variables

In order to evaluate the uncertainty of the bending moment capacity during fire and to calculate the safety level, the uncertainties with respect to the basic variables must be incorporated in the model through their respective probability distribution functions. This modeling of uncertain variables is implemented independently from the deterministic analysis described above, and can therefore easily be modified. This procedure results in a high degree of flexibility and allows for the incorporation of updated information when this is available. The distributional characteristics considered in this paper are based on (Holický and Sýkora, 2010) and the applicable distribution types are presented in Tab. 1. All stochastic variables are considered independent. If more specific information is available with respect to the correlation between specific variables, the developed model can be further adjusted.

Tab. 1: Stochastic variables in full-probabilistic model

Symbol	Name	Distribution
h	beam height	Normal
b	beam width	Normal
f_c(20°C)	20°C concrete compressive strength	Lognormal
f_y(20°C)	20°C yield strength reinforcement	Lognormal
E_c(20°C)	20°C concrete modulus of elasticity	Lognormal
A_s	reinforcement section	Normal
c	concrete cover	bèta [0;3$c_{nominal}$] (*)
χ	model uncertainty	Lognormal

(*) a bèta distribution with the same characteristics as described in (Holický and Sýkora, 2010), but defined over the range [$c_{nominal}$-5;$c_{nominal}$+5] would be more suitable in the opinion of the authors of this paper

Additionally, in order to take the uncertainty regarding the reduction of the mechanical properties at high temperatures into account, a temperature-dependent normal distribution is proposed for the reduction factors for both the concrete compressive strength and the reinforcement yield strength. Both normal distributions are characterized by a mean value equal to the nominal reduction factor of EN 1992-1-2 and a standard deviation based on laboratory tests (Annerel, 2010). For the compressive concrete strength, the standard deviation of the reduction factor is assumed to be 0 at 20°C and 0.045 at 700°C. Linear interpolation is used for intermediate values. For the mechanical reinforcement properties, a similar assumption is made, with a standard deviation of 0.065 at 600°C. The concept is illustrated in Fig. 2 where the 5%, 50% and 95% fractiles of the reduction variable $f_c(\theta)/f_c(20°C)$ are visualized. It is important to note that this uncertainty in reduction factor is additional to the uncertainty on the material characteristics at ambient temperature (20°C). Similarly, Fig. 3 visualizes the 5%, 50% and 95% fractiles of the reduction variable $f_y(\theta)/f_y(20°C)$. No spalling is taken into account, although this could be implemented through a probabilistic degradation function for the concrete cover c.

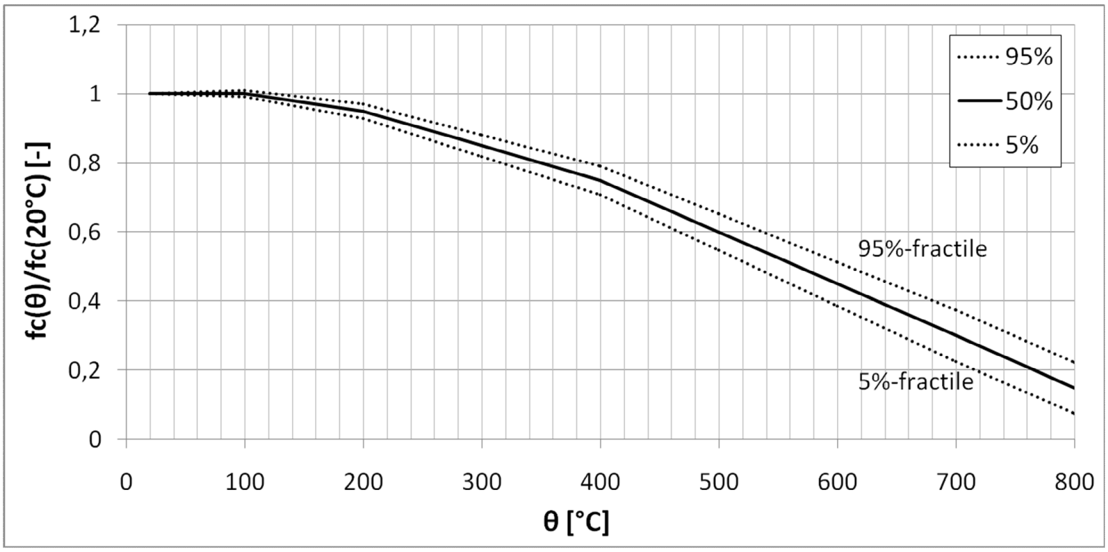

Fig. 2: 5%, 50% and 95% fractiles of reduction factor $f_y(\theta)/f_y(20°C)$

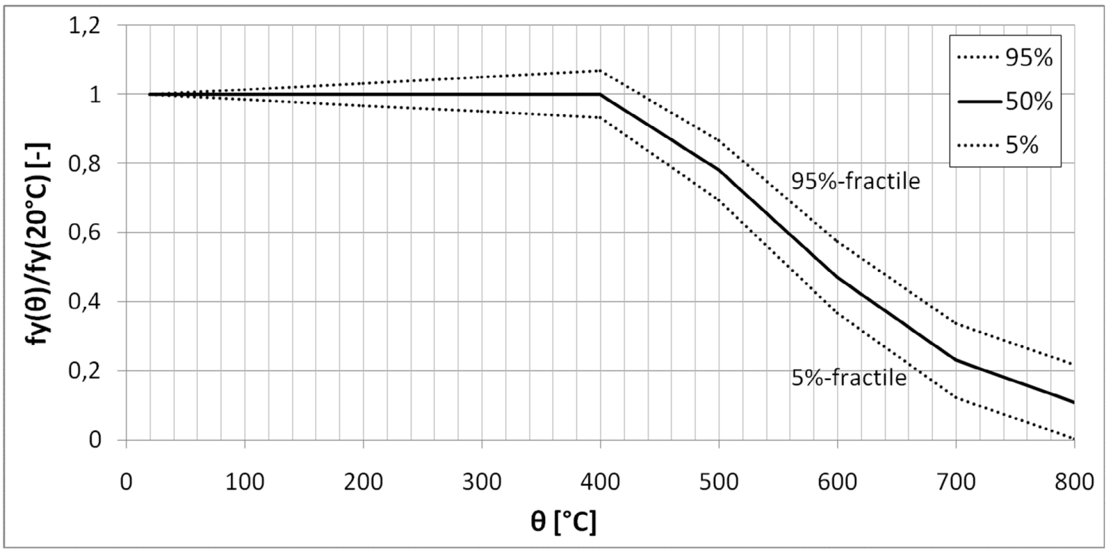

Fig. 3: 5%, 50% and 95% fractiles of reduction factor $f_c(\theta)/f_c(20°C)$

1.3 Uncertainty propagation: crude Monte Carlo simulations

The properties of each beam are characterized by a vector $\underline{X}_i$ of randomly generated values of the variables described above. 10000 vectors $\underline{X}_i$ are generated. For all of these realizations the bending moment capacity during fire is evaluated by the described deterministic analysis. The results of these calculations are analyzed statistically, resulting in an expected value and standard deviation of the bending moment capacity of the concrete beam exposed to fire. This type of analysis by repeated random sampling of the parameter space is generally referred to as Monte Carlo sampling.

2 INTERPRETATION OF MODEL RESULTS

Traditionally, EN 1992-1-2 defines the fire resistance time t_R through equation (2), with $M_{Rd,fi,t}$ the design value of the bending moment capacity and $M_{Ed,fi,t}$ the design value of the bending moment induced by the design loads.

$$M_{Rd,fi,t} \geq M_{Ed,fi,t} \qquad for\ t \leq t_R \tag{2}$$

$M_{Ed,fi}$ is considered to be constant, thus without consideration of indirect thermal actions, i.e. $M_{Ed,fi,t} \equiv M_{Ed,fi}$. A method for evaluating the safety level of a concrete beam during fire is proposed by equations (3) and (4).

$$P_{f,1} = P\left[M_{R,fi,t} < M_{Ed,fi}\right] = \Phi\left(-\beta_1\right) \tag{3}$$

$$P_{f,2} = P\left[M_{R,fi,t} < M_{Rd,fi,t}\right] = \Phi\left(-\alpha_R\beta_2\right) = \Phi\left(-\beta_2\right) \qquad \text{where } \alpha_R = 1 \text{ as } \sigma_E < 0.16\sigma_R \tag{4}$$

Both equations compare the results of the Monte Carlo simulations for the bending moment capacity with the design values of the semi-probabilistic calculation method of EN 1992-1-2. Equation (3) allows for an evaluation of the structural fire resistance, but is dependent on the variable load. On the other hand, as elaborated in (Gulvanessian et al, 2002), equation (4) allows to evaluate the intrinsic safety of the design value of the bending moment capacity of the beam configuration, i.e. β_2 indicates which fractile of the bending moment distribution corresponds to the design value given by the Eurocodes. According to Eurocode 0 (CEN, 2002), the sensitivity factor α_R can be assumed equal to 1 for this case since $\sigma_E/\sigma_R < 0.16$. This result is based on additional Monte Carlo simulations in which the standard deviation of the design value of the bending moment induced by the design loads was simulated and compared to the simulated standard deviation of the bending moment capacity. The stochastic characteristics for the design loads are based on (Holický and Sýkora, 2010).

Although β_1 and β_2 are not the conventional definitions of the safety index β and consider only the stochastic nature of the resistance effect, both equations allow to investigate the influence of the basic variables on the safety level. Since the main objective of this study is to compare the safety level of different configurations and to analyze the effect of basic stochastic assumptions on the evolution of the safety level, the deviation from the classical definition is acceptable. Furthermore, since α_R can be assumed equal to 1, at the fire resistance time t_R equations (3) and (4) are equal, as shown mathematically by equation (5). As such, the fire resistance time of the beam can be approximated by the intersection of the β_1 and β_2 curves.

$$\Phi\left(-\beta_{1,t_R}\right) = P\left[M_{R,fi,t_R} < M_{Ed,fi}\right] = P\left[M_{R,fi,t_R} < M_{Rd,fi,t_R}\right] = \Phi\left(-\alpha_R\beta_{2,t_R}\right) = \Phi\left(-\beta_{2,t_R}\right) \tag{5}$$

Finally, both equations (3) and (4) can be evaluated by using the frequentist interpretation of probability.

3 APPLICATION EXAMPLE

As an application example , simulation results are presented for the concrete beam presented in Table 2. The distribution parameters for the random variables considered are given in Tab. 3, based on (Holický and Sýkora, 2010). According to the ‘table method’ of (CEN, 2004a) the characteristics of the example beam correspond to a fire resistance of 90 min (a = 40 mm, a_{sd} = 50 mm, b_{min} = 300 mm) when the beam is simply supported. In accordance with the calculation methodology of EN 1992-1-1 (CEN, 2004b) the example beam has a design value of 358 kNm for the bending moment capacity at ambient temperature.

Table 2: Configuration example concrete beam

Symbol	Characteristic	Unit	Nominal Value
h	beam height	mm	600
b	beam width	mm	300
f_{ck}(20°C)	20°C characteristic compressive strength	MPa	40
f_{yk}(20°C)	20°C characteristic yield strength	MPa	500
E_c(20°C)	20°C concrete modulus of elasticity	GPa	34.5
E_s(20°C)	20°C steel modulus of elasticity	GPa	200
c_1	bottom concrete cover	mm	30
$Ø_1$	bottom reinforcement diameter	mm	20
$\#_1$	number of bottom reinforcement bars	-	5
s_1	spacing bottom reinforcement bars	mm	50
c_2	top concrete cover	mm	30
$Ø_2$	top reinforcement diameter	mm	20
$\#_2$	number of top reinforcement bars	-	5
s_2	spacing top reinforcement bars	mm	50
χ	model uncertainty	-	1.2

Tab. 3: Stochastic models for variables

Symbol	Variable	Distribution type	Mean μ	Standard deviation σ
h	beam height	normal	600 mm	5 mm
b	beam width	normal	300 mm	5 mm
f_c(20°C)	20°C concrete compressive strength	lognormal	45.4 MPa	2.7 MPa
f_y(20°C)	20°C steel yield strength	lognormal	581 MPa	41 MPa
E_c(20°C)	20°C concrete modulus of elasticity	lognormal	34.5 GPa	5.2 GPa
$c_{1,2}$	concrete cover	bèta [0;3$c_{nominal}$]	30 mm	2 mm
χ	model uncertainty	lognormal	1.2	0.15

Simulation results for the example beam are presented in Fig. 4. The curves in Fig. 4 are normalized according to the 50% fractile of the bending moment capacity ($M_{R,fi,t,50}$) at the start of the fire (i.e. at ambient temperature). The intersection of the design value of the bending moment capacity and the design value of the bending moment induced by the design loads is situated at approximately 83 minutes of exposure.

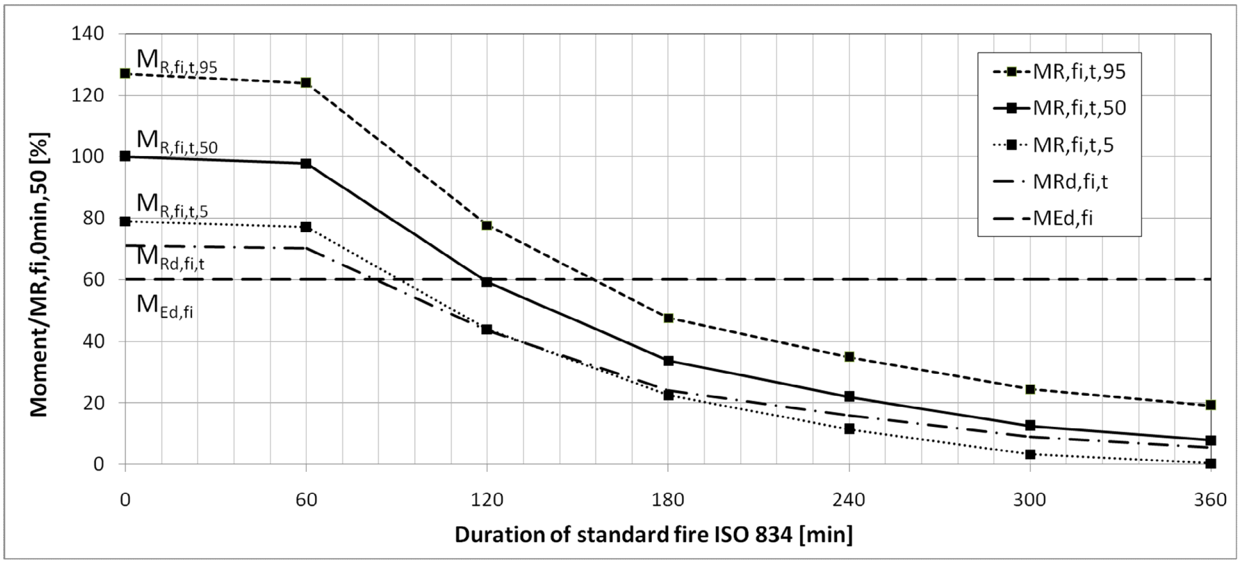

Fig. 4: Model results for example beam ($M_{R,fi,t,50}$ = 583 kNm)

The calculated safety indices β_1 and β_2 according to equation (3) and (4) are visualized in Fig. 5. The intersection of both curves corresponds to a fire resistance of 80 min. While the decrease of β_1 indicates the increasing probability of structural failure, the decrease of β_2 corresponds to an increasing probability that the design value of the bending moment capacity ($M_{Rd,fi,t}$) overestimates the actual bending moment capacity ($M_{R,fi,t}$) of the example beam. The latter can be explained by the uncertainty related to the reduction of material properties at elevated temperatures and the uncertainty of the reinforcement temperature (i.e. the concrete cover). These elements are not explicitly taken into account by the semi-probabilistic design methods in the Eurocode (CEN, 2004a).

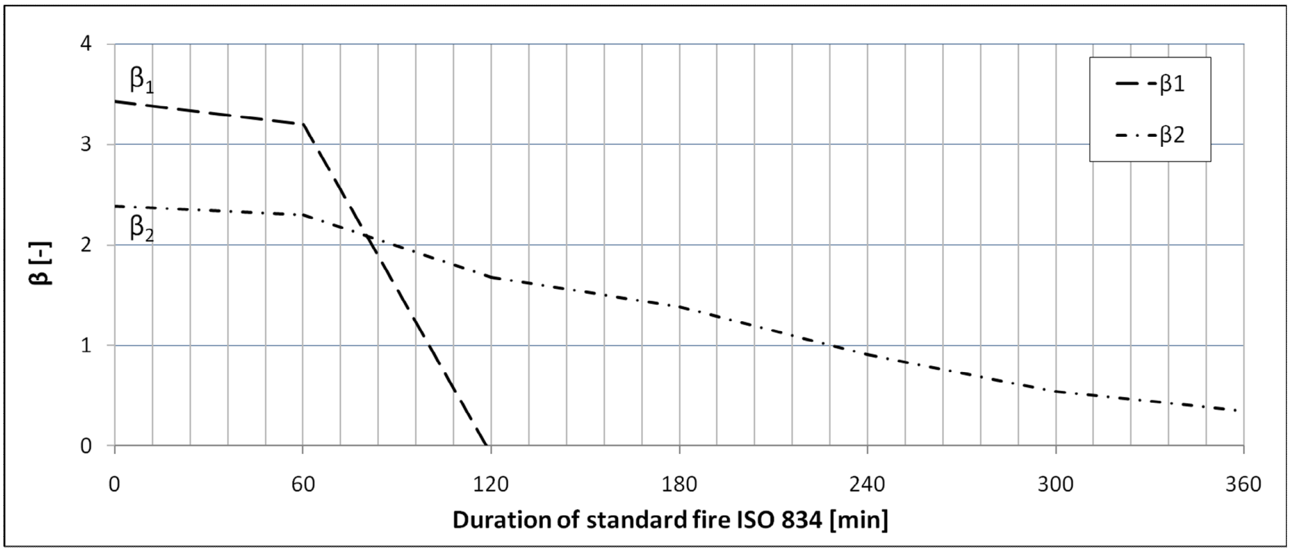

Fig. 5: β_1 and β_2 for example beam configuration

By altering the beam configuration the simulated fire resistance time can be increased. Simulations indicate that an increase of the concrete cover is particularly efficient, while increasing the concrete compressive strength has less impact.

CONCLUSIONS

- A full-probabilistic model is developed for analyzing the safety level of concrete beams
- Based on the probabilistic analysis of a beam the fire resistance time was found to be smaller than tabulated by EN 1992-1-2 (CEN, 2004a).
- The fire resistance of a beam can be increased by altering the beam configuration, or by decreasing the uncertainty on e.g. the concrete cover through sampling and testing.

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